The Statistical Guide to a Silver Buckle

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August 20, 2017

Abstract

In this paper, we analyze the results of the 2017 Western States 100 Mile Endurance Run. We build a simple classification model which estimates each runner's likelihood of finishing under 24 hours and earning a coveted silver buckle. Using this model, we establish an empirical 24-hour pace and compare it to the official pacing guide. We also calculate a silver buckle "frontier" pace which represents the largest deficit from which it is still feasible to recover. We also group runners by eventual finishing status and examine their performances relative to our model projections. From this we draw several conclusions about how best to pace oneself to a Western States silver buckle. In particular, we find that silver buckle winners consistently accelerate their pace while non-winners do not. Lastly, we examine several categories of noteworthy performances including biggest comebacks, biggest blowups and steadiest pacing.

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Introduction

The Western States Endurance Run is a 100-mile footrace following the Western States trail from Squaw Valley to Auburn, California. It is held annually in late June and is the oldest and most prestigious 100-mile running race in the world. Special recognition is given to those finishing the race under 24 hours: a silver belt buckle. Those finishing in more than 24 hours and under the 30-hour cutoff are awarded bronze buckles. For many runners, earning the silver buckle is the culmination of years of preparation and training.

***Image 1:*** *The Western States silver buckle*

Additional recognition is given to the top ten male and female finishers. In addition to the silver buckle, they are automatically invited to return to the race next year, thereby avoiding the entry lottery. In this paper, we will often break out these runners as a separate category for informational purposes. We will also break out those runners that did not finish the race (DNF) either by choice or by failing to reach a particular aid station by the cutoff time.

Proper pacing is key to a successful race, particularly for a race as long and challenging as Western States. Some runners start fast and try to hang on all the way to Auburn. Some start conservatively and try to accelerate as the race unfolds. Some try to maintain an even pace all day. In this paper, we will use the actual results of the 2017 race to estimate the ideal silver-buckle pace. Using this estimated pace, we will also examine how various groups of runners paced themselves in their quest to earn the coveted silver buckle.

Along the course there are a total of 21 aid stations, in addition to the start and finish lines. The website *ultralive.net* collected live timing of runners at 18 of the 21 aid stations, plus the start and finish. This paper uses that data and gratefully acknowledges their contribution to the race. This analysis would not be possible otherwise.

Initial Results

To begin our investigation, let's look at some high-level results from the race. Chart 1 shows the elapsed time of each runner to each of the 18 aid stations and finish line. Here we have grouped the runners into four categories: "Top 10" (all of which were under 24 hours), "Sub-24 Hours" (which are those that were not top 10 but still under 24 hours), "24+ Hours" (which are those that finished between 24 and 30 hours) and "DNF" (which are those that failed to finish). The results are as one might expect: the field spreads out as the race progresses, with the faster runners slowly pulling away from the slower ones. However, as we will see, there is actually significantly more happening when we look closer.
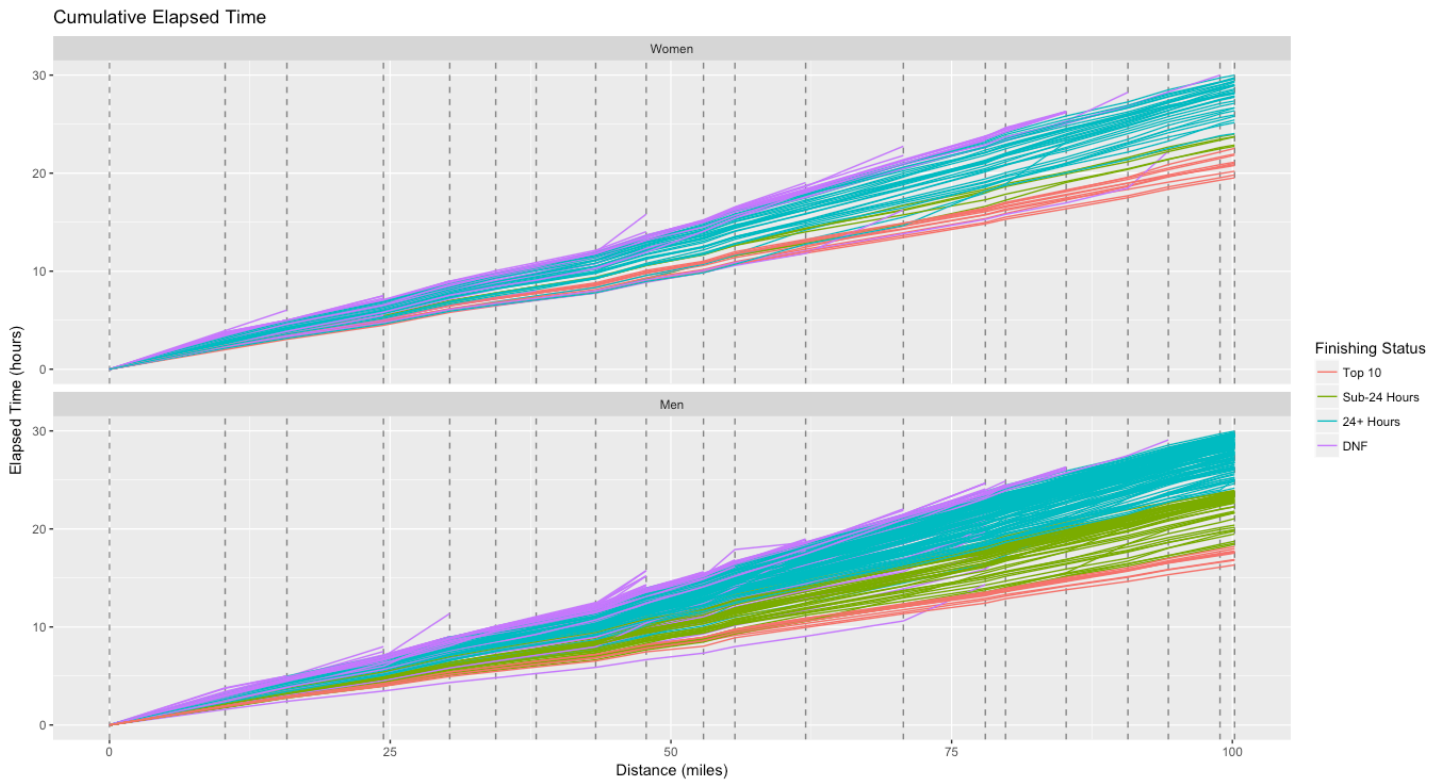
Chart 1: *Cumulative time by gender and finishing status*

Table 1 shows the breakdown of finishing status for all 369 starters. Table 2 shows the same breakdown but in percentage terms. Overall, a total of 76 silver buckles were awarded, representing about 20% of the starters. Of the 80% who failed to earn a silver buckle, 47% finished over 24 hours (earning the bronze buckle) and 33% did not finish.

	Female	Male	Total
Top 10	10	10	20
Sub-24 Hours	5	51	56
24+ Hours	31	141	172
DNF	39	82	121
Total	85	284	369

Table 1: *Finishing status count by gender*

	Female	Male	Total
Top 10	12%	4%	5%
Sub-24 Hours	6%	18%	15%
24+ Hours	36%	50%	47%
DNF	46%	29%	33%
Total	100%	100%	100%

Table 2: *Finishing status proportion by gender*

Official Silver Buckle Pace

The Western States race organizers publish an official 24-hour pace chart, shown in Table 3. As we will see, this official pace is generally reasonable but in some segments does not accurately capture the actual pacing of silver buckle winners.

Aid Name	Aid Distance	Official Pace
Start	0.0	0:00
Lyon Ridge	10.3	2:10
Red Star Ridge	15.8	3:20
Duncan Canyon	24.4	5:00
Robinson Flat	30.3	6:30
Miller's Defeat	34.4	7:15
Dusty Corners	38.0	7:55
Last Chance	43.3	8:55
Devil's Thumb	47.8	10:15
El Dorado Creek	52.9	11:20
Michigan Bluff	55.7	12:20
Foresthill	62.0	13:45
Peachstone	70.7	15:45
Rucky Chucky	78.0	17:40
Green Gate	79.8	18:20
Auburn Lake Trails	85.2	19:50
Quarry Road	90.7	21:10
Pointed Rocks	94.3	22:20
Robie Point	98.9	23:40
Finish Line	100.2	24:00

*Table 3: Finishing status proportion by gender*

Can we use the actual results from the 2017 event to build a more accurate pace chart? The next section will do just that using the observed results of all 369 starters in the 2017 event.

Building a Model for Silver Buckle Pace

For any given aid station, we have a record of each runner's cumulative time and their ultimate finishing status (silver buckle or not). We can ask: given a runner's cumulative time to that point, how likely are they to ultimately earn a silver buckle? This type of problem is well-suited to a simple logistic regression model. Logistic models take a set of inputs (in our case just cumulative time) and output the likelihood of observing one of two binary states (in our case earning a silver buckle or not).

Let's look at a specific example. We'll look at the Foresthill aid station, which is mile 62. A total of 301 runners reached this aid station. Of those 301 runners, 76 went on to earn a silver buckle. The other 225 runners did not (171 finished in more than 24 hours and 54 did not finish). Ideally, there would be a single point such that all runners who arrived at Foresthill prior to that point earned a silver buckle and all those who arrived after did not. Of course, reality is not so simple. There were many fast runners who arrived at Foresthill early yet failed to finish (Jim Walmsley is one example). There were also many runners who arrived at Foresthill relatively slowly yet rallied hard to finish under 24 hours. (We will examine several of these examples later.)

Chart 2 below shows each runner as a single point. Their position on the x-axis is their cumulative time upon reaching Foresthill. Their position on the y-axis is either 0 (failed to earn a silver buckle) or 1 (earned a silver buckle). (Note that the points have been vertically "jittered" by a small amount to more clearly display otherwise overlapping points.) The logistic regression model essentially constructs a backwards "S" shaped curve which assigns a probability to each point of being in the silver-buckle category.
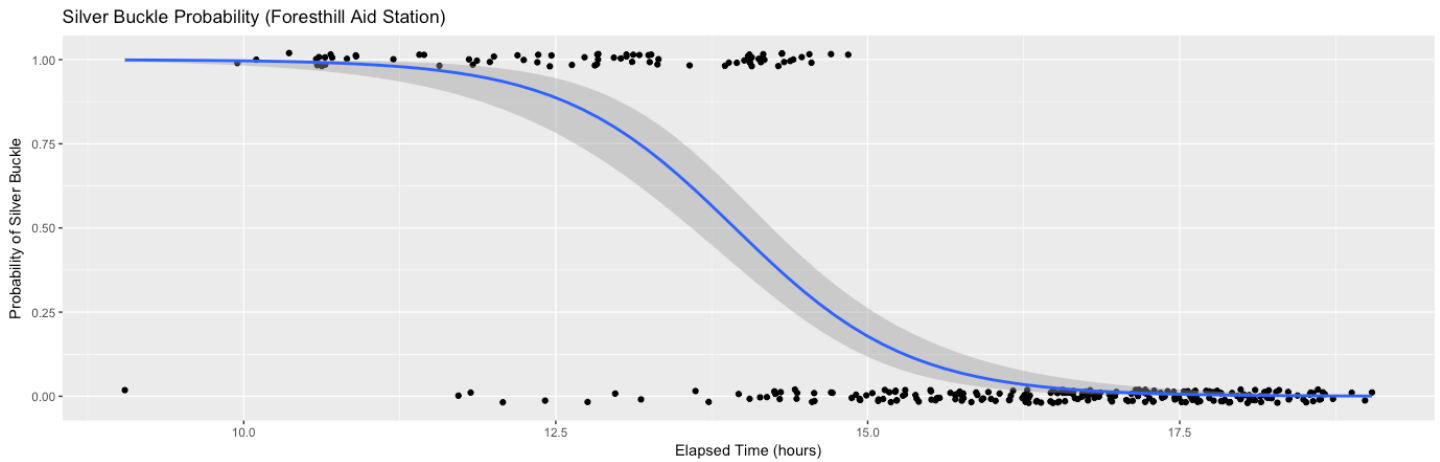
Chart 2: *Logistic regression model for Foresthill aid station.*

As you can see, the model does a good job for those runners reaching Foresthill in less than 12.5 hours (assigning close to 100% likelihood) and those runners arriving in more than 15 hours (assigning close to 0% likelihood). Between 12.5 and 15 hours, things are not as clean but the model is fit to minimize the total error rate.

Given this model, we can look for the point at which the model assigns a 50% probability. In the case of Foresthill, this occurs at 13 hours and 56 minutes. This means that if a large (theoretical) group of runners arrived at Foresthill at exactly 13:56, about half would eventually earn a silver buckle and half would not. We can therefore take 13:56 as the "empirical" 24-hour silver-buckle pace to this aid station. We can then repeat this process for each of the 18 aid stations for which we have runner data and construct a complete 24-hour pace chart.

Model Results

Table 4 compares our model's empirical silver-buckle pace and the official pace. You can see that they generally agree for the first 62 miles of the race. Beyond that, however, our empirical model actually differs from the official model by more than 30 minutes later in the race. As an example, while the official pace at Quarry Road (90.7 miles) is 21:10, our empirical model still assigns a 50% probability of a silver buckle for a runner arriving at 21:43, a full 32 minutes later (and less than 10 miles from the finish).

Aid Name	Aid Distance	Official Pace	Empirical Pace	Difference
Start	0.0	0:00	0:00	0:00
Lyon Ridge	10.3	2:10	2:16	0:05
Red Star Ridge	15.8	3:20	3:26	0:05
Duncan Canyon	24.4	5:00	4:59	-0:01
Robinson Flat	30.3	6:30	6:27	-0:03
Miller's Defeat	34.4	7:15	7:12	-0:02
Dusty Corners	38.0	7:55	7:50	-0:05
Last Chance	43.3	8:55	8:45	-0:10
Devil's Thumb	47.8	10:15	10:09	-0:05
El Dorado Creek	52.9	11:20	11:20	0:00
Michigan Bluff	55.7	12:20	12:19	-0:01
Foresthill	62.0	13:45	13:56	0:11
Peachstone	70.7	15:45	16:07	0:21
Rucky Chucky	78.0	17:40	18:02	0:21
Green Gate	79.8	18:20	18:45	0:25
Auburn Lake Trails	85.2	19:50	20:14	0:23
Quarry Road	90.7	21:10	21:43	0:32
Pointed Rocks	94.3	22:20	22:43	0:22
Robie Point	98.9	23:40	23:46	0:06
Finish Line	100.2	24:00	24:00	0:00

Table 4: *Official and empirical pace chart*

Chart 3 shows these differences in graphical form. Relative to the empirical pace, the official pace appears to be about 10 minutes too slow at Last Chance (mile 43.3) and consistently 20-30 minutes too fast from Peachstone (70.0) to Pointed Rocks (94.3). In other words, for a runner who finds themselves slipping behind the official 24-hour pace after Foresthill, do not despair! Evidence suggests you have better than a 50% chance of rallying to that silver buckle if you can keep within about 20 minutes of the official pace.
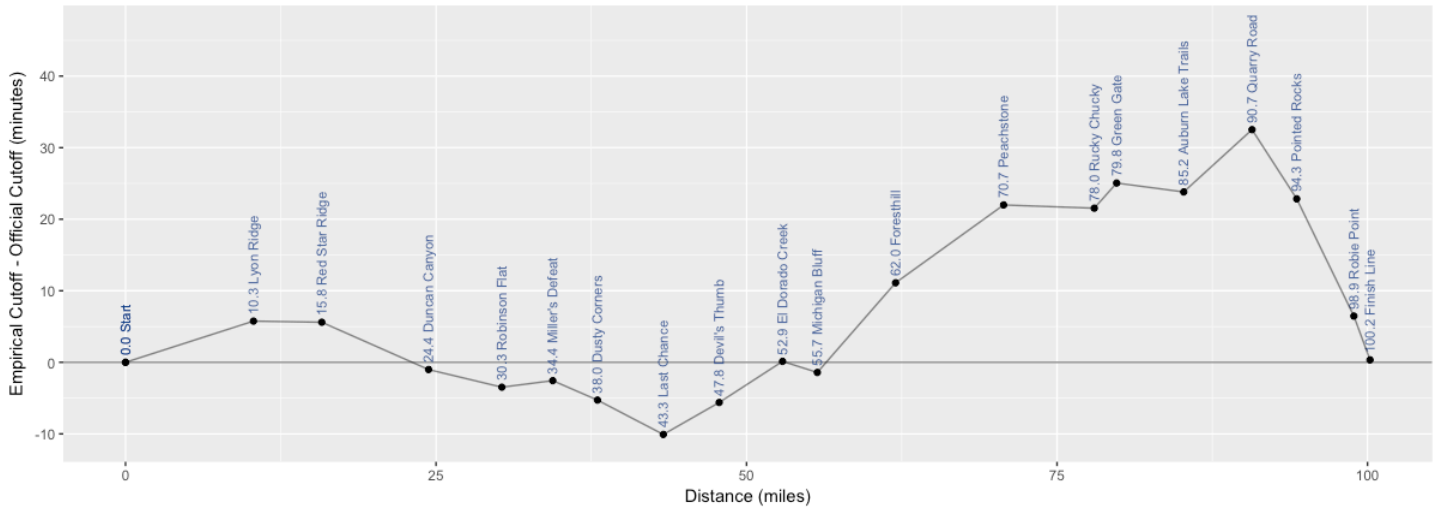
Chart 3: *Difference between empirical 24-hour pace and official pace*
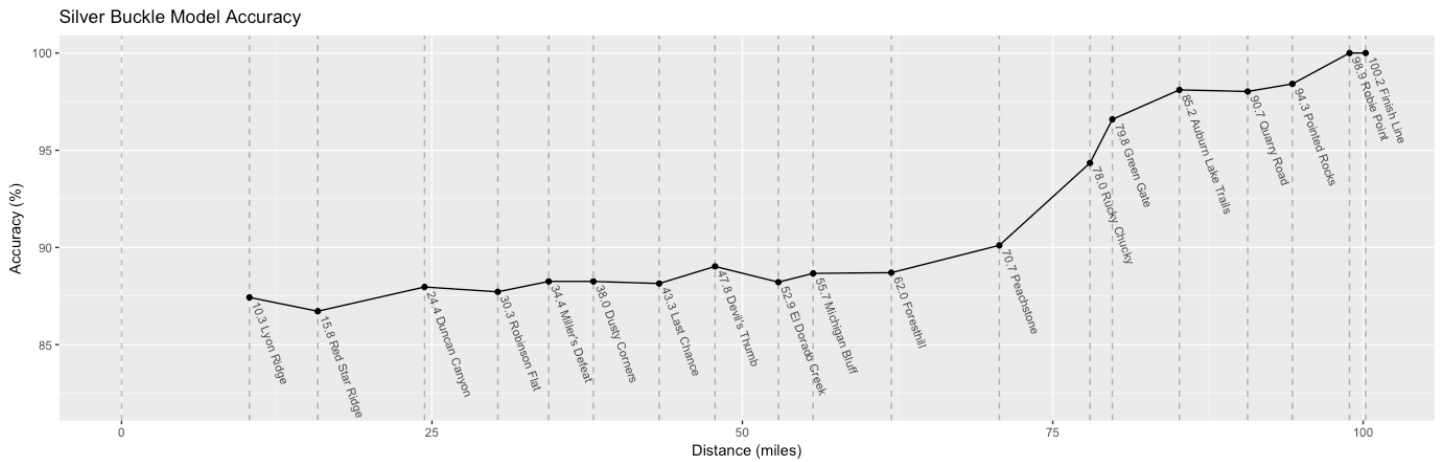
Evaluating Model Accuracy

Now that we have a model which outputs a likelihood of earning a silver buckle for each runner at a particular aid station, we can evaluate its accuracy. We will do this in a very simple manner: if the model assigns a likelihood above 50% for a particular runner, we will categorize that runner as "predicted silver buckle". If the model assigns a likelihood below 50%, we will categorize that runner as "predicted no silver buckle". We then have the following for possibilities of model outcome:

|		Actual Outcome	
		Silver Buckle	No Silver Buckle
Model Prediction	Silver Buckle	True Positive	False Positive
	No Silver Buckle	False Negative	True Negative

Our accuracy will then be given by:

$$Accuracy = \frac{True\ Positives + True\ Negatives}{Total\ Observations}$$

Chart 4 shows our model's accuracy for each aid station. Interestingly, the model is actually quite accurate, even for early aid stations. In particular, our model correctly categorized over 87% of all runners who arrived at the Lyon Ridge aid station, just 10.3 miles into the race. While the accuracy hovers around 87-90% for the first 70 miles, it rises sharply to the mid-90% range by Rucky Chucky (mile 78) and Green Gate (mile 79.8). By Auburn Lake Trails (mile 85.2), the model is correctly categorizing over 97% of all runners correctly. At Robie Point (mile 98.9) the model is fully 100% accurate. Of course, it doesn't take much effort to tell who is going to break 24 hours when you're only 1.3 miles from the finish!

Chart 4: *Model accuracy by aid station*

Now that we have built a model for empirical 24-hour pace, let's revisit Chart 1 and overlay this pace. Chart 5 shows the result. It appears that the model did a decent job separating the silver buckle winners (pink and green) from the non-silver buckle winners (blue and purple). It also appears that there were a handful of silver buckle winners who were behind 24-hour pace for a significant portion of the race but rallied to finish under 24 hours. We will investigate these runners in more detail later.
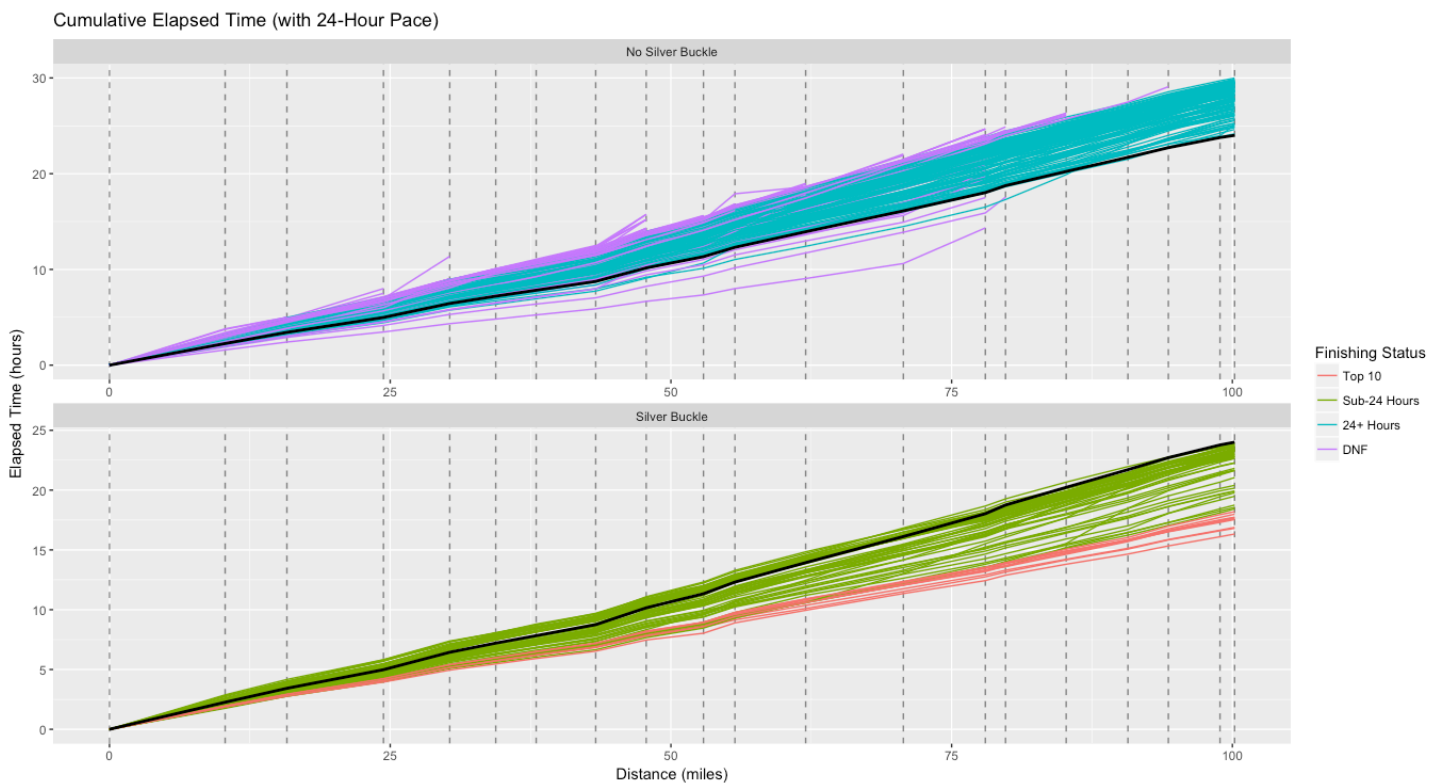

Chart 5: *Cumulative elapsed time with empirical 24-hour pace*

Empirical Silver Buckle Pace by Segment

Now that we have built a model of the empirical, cumulative 24-hour pace to each aid station, we can examine each segment and compute the model pace over that portion of the course. Chart 6 shows the 19 segments and the pace required in order to stay on silver buckle pace.
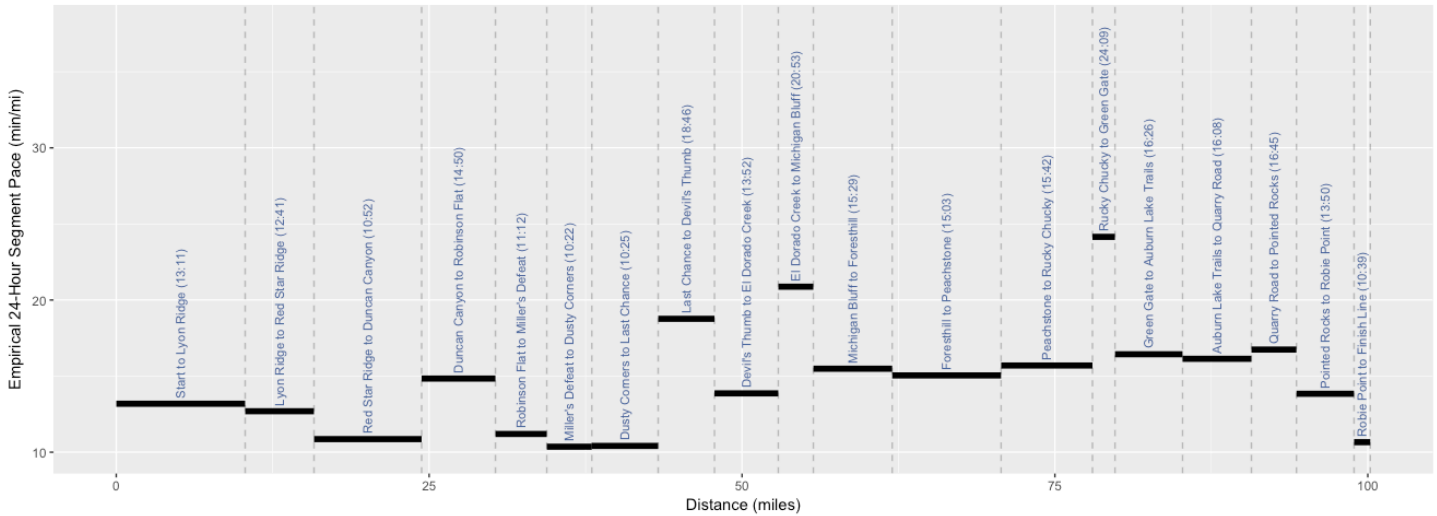
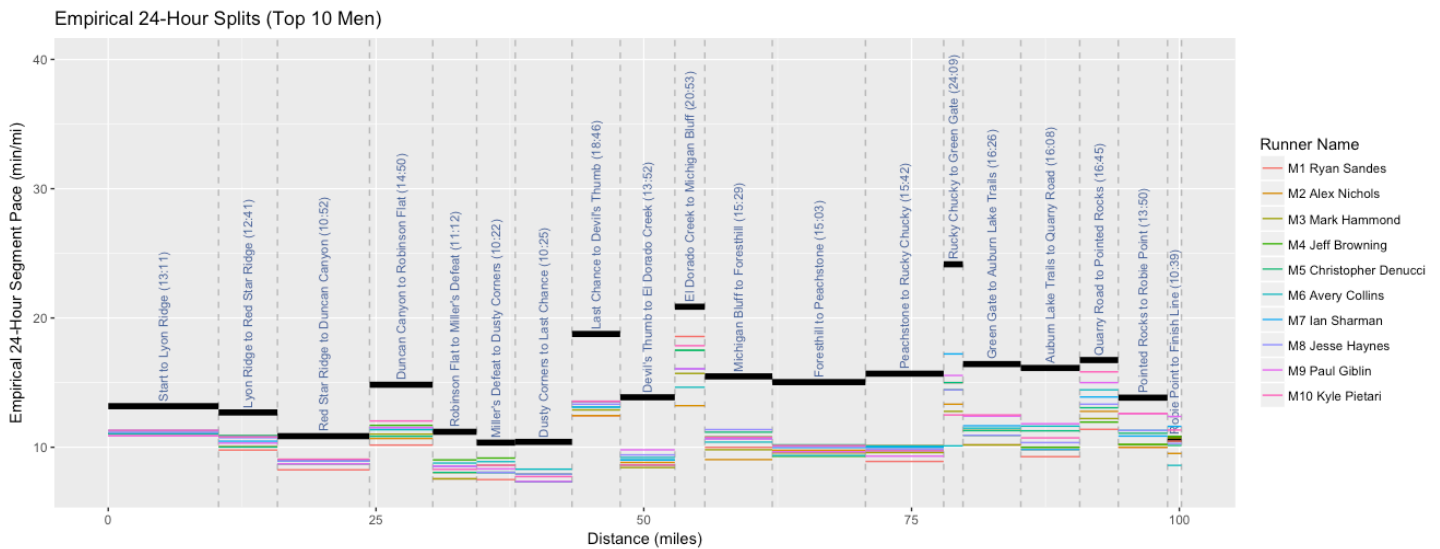***Chart 6:** Empirical model pace by segment*

Not surprisingly, the pace varies widely throughout the race. While the overall silver buckle pace works out to 14:22 minutes per mile, there are some segments which are significantly slower. In particular, the climb from the Rucky Chucky river crossing to Green Gate (mile 78 to 79.8) is a 24:09 min/mile pace. Likewise, the climb from El Dorado Creek to Michigan Bluff (mile 52.9 to 55.7) is a 20:53 min/mile pace. On the flip side, the final mile from Robie Point to the finish (mile 98.9 to 100.2) is a 10:39 min/mile pace. The segments from Miller's Defeat to Dusty Corners (mile 34.4 to 38) and from Dusty Corners to Last Chance (mile 38 to 43.3) are also quite fast at 10:22 min/mile and 10:25 min/mile, respectively.

Segment	Segment Distance	Cumulative Distance	Segment Pace (min/mile)	Cumulative Pace (min/mile)
Start to Lyon Ridge	10.3	10.3	13:11	13:11
Lyon Ridge to Red Star Ridge	5.5	15.8	12:41	13:01
Red Star Ridge to Duncan Canyon	8.6	24.4	10:52	12:15
Duncan Canyon to Robinson Flat	5.9	30.3	14:50	12:45
Robinson Flat to Miller's Defeat	4.1	34.4	11:12	12:34
Miller's Defeat to Dusty Corners	3.6	38.0	10:22	12:22
Dusty Corners to Last Chance	5.3	43.3	10:25	12:07
Last Chance to Devil's Thumb	4.5	47.8	18:46	12:45
Devil's Thumb to El Dorado Creek	5.1	52.9	13:52	12:51
El Dorado Creek to Michigan Bluff	2.8	55.7	20:53	13:16
Michigan Bluff to Foresthill	6.3	62.0	15:29	13:29
Foresthill to Peachstone	8.7	70.7	15:03	13:41
Peachstone to Rucky Chucky	7.3	78.0	15:42	13:52
Rucky Chucky to Green Gate	1.8	79.8	24:09	14:06
Green Gate to Auburn Lake Trails	5.4	85.2	16:26	14:15
Auburn Lake Trails to Quarry Road	5.5	90.7	16:08	14:22
Quarry Road to Pointed Rocks	3.6	94.3	16:45	14:27
Pointed Rocks to Robie Point	4.6	98.9	13:50	14:25
Robie Point to Finish Line	1.3	100.2	10:39	14:22

***Table 5:** Empirical segment pace*

Table 5 shows the segment pace and cumulative pace in minutes/mile for each of the 19 segments.

Chart 7 is the segment pace chart with pace of each of the top ten men overlaid. Perhaps not surprisingly, all ten men ran every segment under 24-hour pace (with the exception of the final mile from Robie Point to the finish). Of particular note, all ten men ran more than 5:00 min/mile under silver-buckle pace from Foresthill (mile 62) to the Rucky Chucky river crossing (mile 78). These guys are really fast!

*Chart 7: Empirical model pace by segment with Top 10 men*

Chart 8 shows the same for the top ten women. Unlike the men, however, we can see that several ladies began the race conservatively and accelerated later. In fact, six of the eventual top ten women would run the first segment from the start to Lyon Ridge (mile 10.3) behind silver-buckle pace.
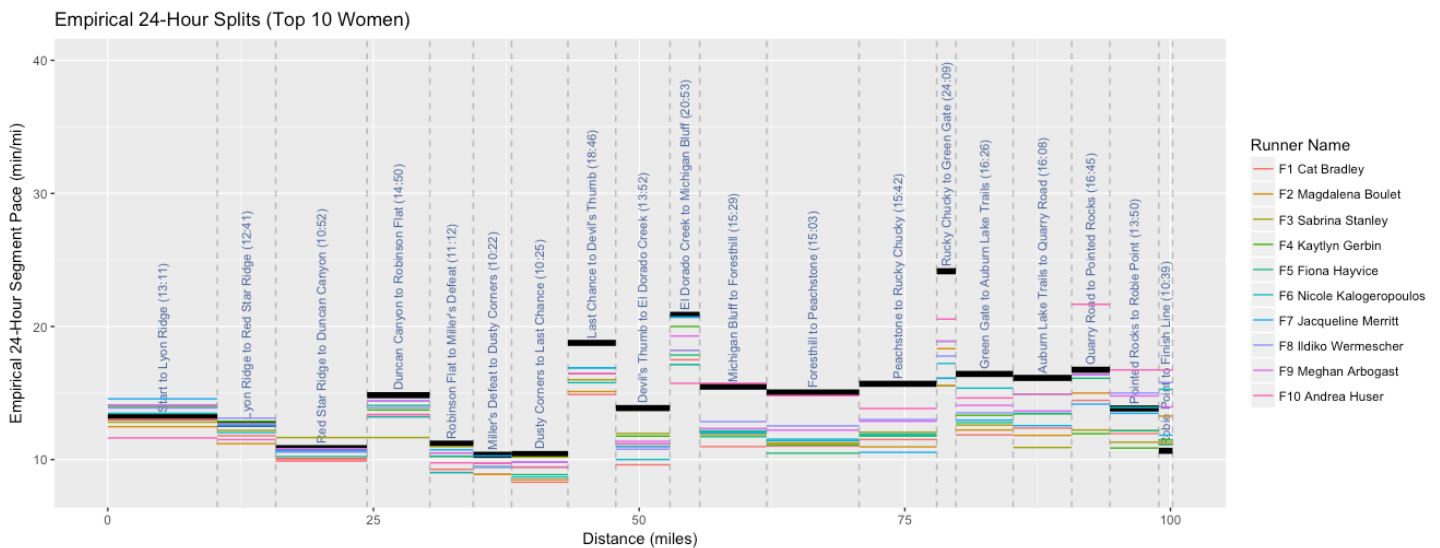
Chart 8: Empirical model pace by segment with Top 10 men

Aid Station Pace Chart

Chart 9 shows the relationship between each runner's elapsed time and their final finishing time at each aid station. We have assigned a finishing time of 30:00 to all runners who failed to finish (DNF). In each chart, the horizontal line represents a 24-hour finish. The vertical line represents the empirical silver buckle pace to the given aid station. The points are colored by final finishing status.

Aid Station Pace Chart
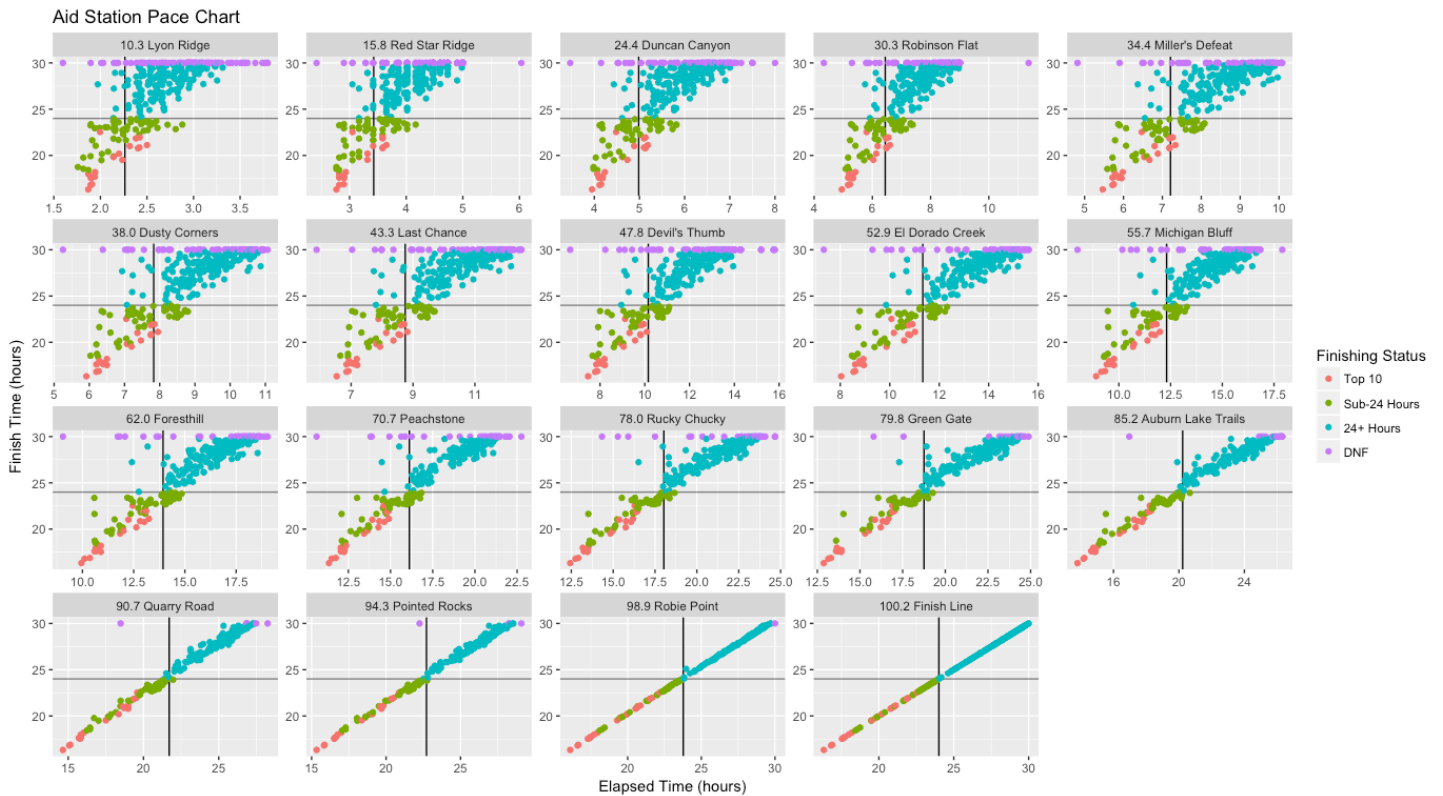
Chart 9: Relationship between elapsed time to aid station and evetual finishing time

These charts provide a method of visually inspecting the model accuracy that we computed earlier. In particular, the more accurate the model, the better it will separate the silver buckle winners (Top 10 and Sub-24 Hours) to the left of the vertical line and the non-silver buckle winners (24+ Hours and DNF) to the right of the line. You can see that the early aid stations still contain a good bit of mixing, but the two categories sort themselves out as the race progresses.

Silver Buckle Probabilities

Recall that at each aid station our model assigned each runner a likelihood of being an eventual silver buckle winner based on their elapsed time to that point. Therefore, by stringing together these model estimates, we can observe how each runner's race unfolded, as measured by silver buckle likelihood.

Chart 10 shows this likelihood for each runner throughout the race, grouped by their final finishing status. The latest point any of the twenty male and female Top 10 finishers had less than a 50% chance of a silver buckle was Devil's Thumb (mile 47.8). In fact, all of the Top 10 men were above 80% for the entire race.

The Sub-24 Hours finishers are more interesting. While many got off to a strong start and stayed well above 50% throughout the race, there were also many that started slowly and remained under 50% likelihood until quite late in the race. However, with one exception (which we will examine in more detail later), all of the eventual silver buckle winners would rally above the 50% point by Auburn Lake Trails (mile 85.2).
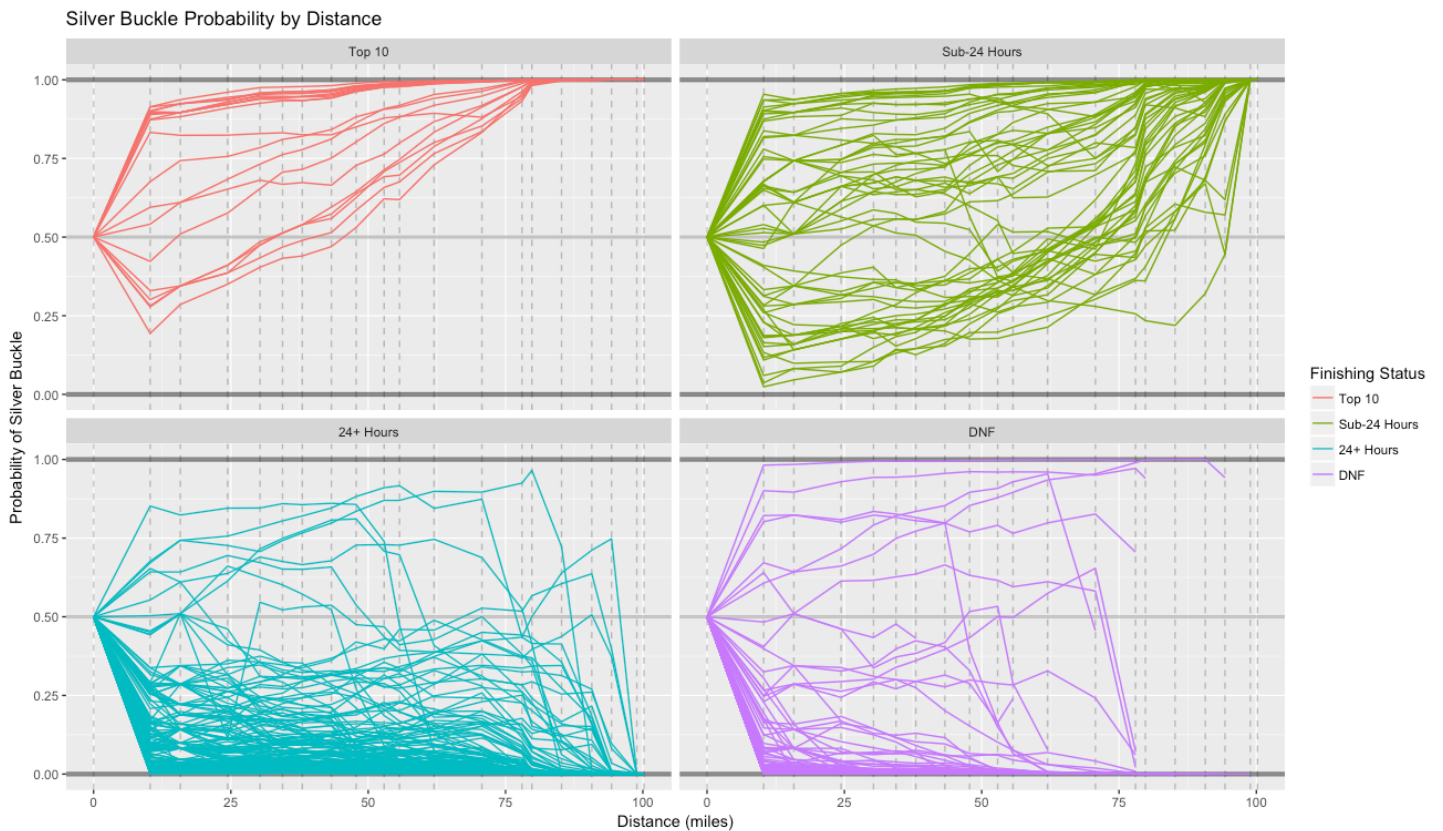
Chart 10: *Individual runner silver buckle likelihood by aid station*

On the other hand, the large majority of 24+ Hour finishers never surpassed 50% likelihood at any point in the race. In fact, only 11 of the 172 runners finishing in more than 24 hours exceeded 50% likelihood at any point. Furthermore, all but 6 of the 172 were behind pace right from the first aid station, Lyon Ridge (mile 10.3).

Lastly, the DNFs were more of a mixed bag. Many of the DNFs had silver buckle likelihood of less than 10% at the first aid station and never recovered. On the other hand, there were 9 runners with likelihood above 50% who would nevertheless fail to finish the race.

Accelerating Towards a Silver Buckle

Now that we have established an empirical silver buckle pace, let's evaluate each runner's cumulative pace relative to this threshold. Do silver buckle winner get out to a fast start and simply hang on? Or do they start slowly and accelerate?

Chart 11 shows the cumulative pace (measured in minutes per mile) relative to our model 24-hour pace for men. The Top 10 men began fast (about 2 minutes per mile ahead of pace) and slowly accelerated throughout the race, ending ahead of pace by 3.5 to 4.5 minutes per mile.

The Sub-24 Hour male finishers also accelerated throughout the race, though they began with a very wide distribution of initial paces – there was a difference of more than 6 minutes per mile from the Start to Lyon Ridge (mile 10.3) among the silver buckle winners. While this wide dispersion persisted throughout the race, as a group the silver buckle winners slowly accelerated towards Auburn.
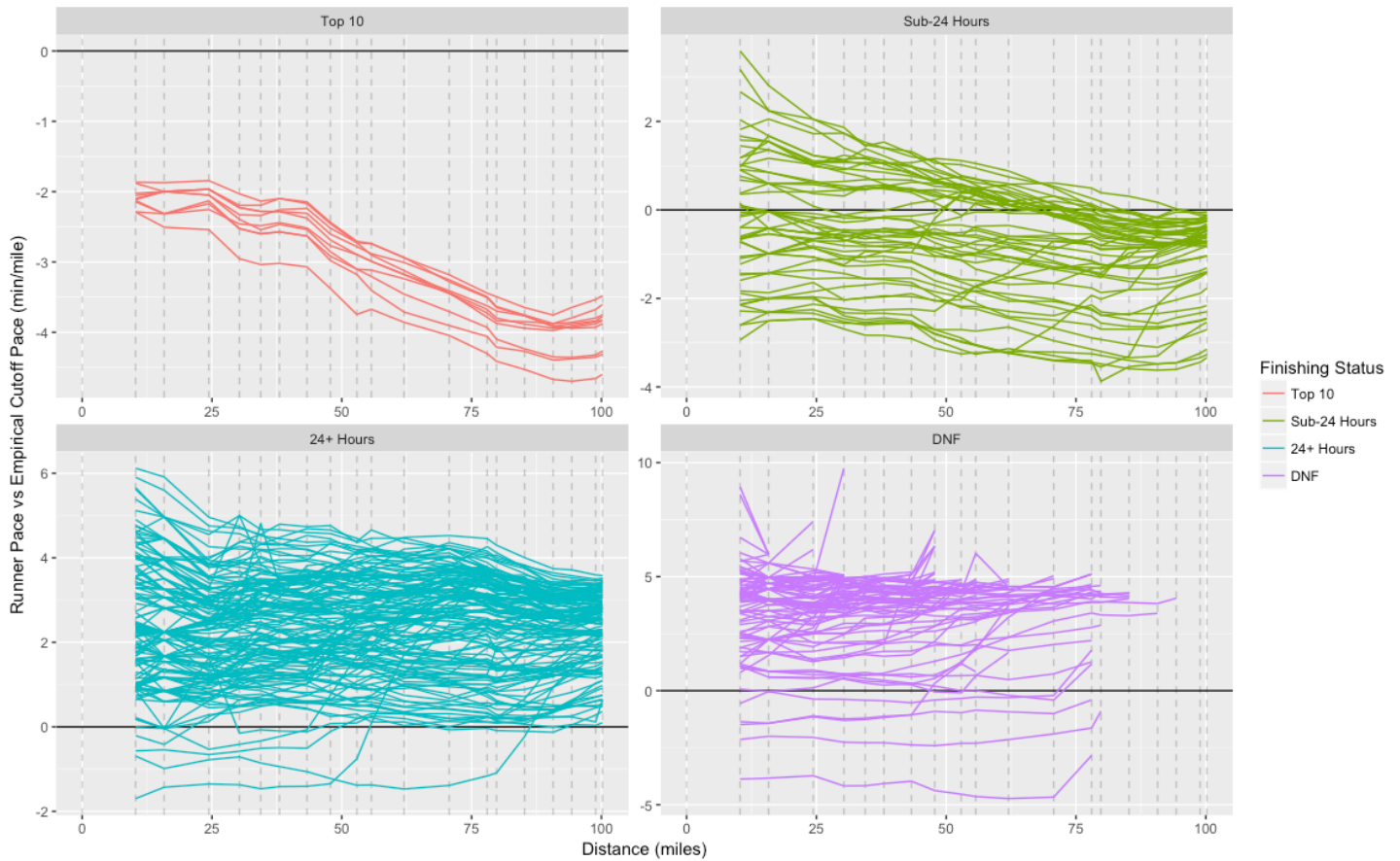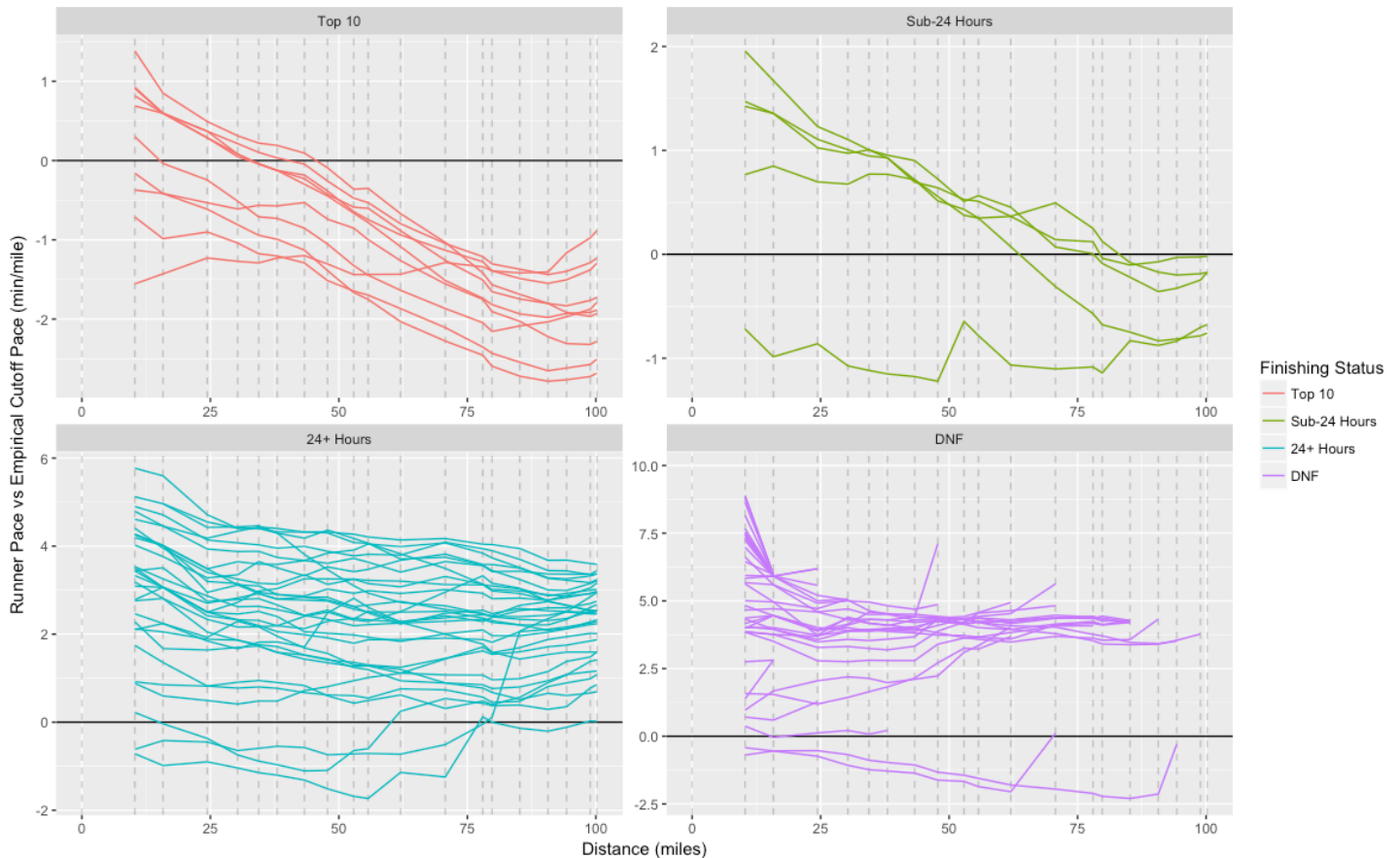
Chart 11: *Cumutative pace (min/mile) for men*

On the other hand, the 20+ Hour finishers and DNFs largely remained flat in their pacing relative to silver buckle pace. A handful of runners got off to fast starts and later blew up, but the overwhelming majority of non-silver buckle winners started slowly and simply maintained their pace.

Chart 12 shows the same for female runners. Again, the same pattern presents itself: silver buckle winners accelerate (relative to 24-hour pace) throughout the race and non-silver buckle winners do not. In the case of the women, 9 of the 15 women who earned silver buckles were actually behind pace at the first aid station, but slowly accelerated to solid finishes.

Chart 12: *Cumutative pace (min/mile) for men*

Chart 13 takes the same data and normalizes each runner's speed by their average deviation from 24-hour pace. This allows us to see clearly how each runner is speeding up or slowing down, relative to their average deviation from 24-hour pace. In other words, we can focus on changes in pacing, without regard for the absolute speed of each runner. To clarify, we are not measuring changes in a runner's absolute pace. As Chart 5 showed, this varies considerably for each segment of the course. Instead, we are measuring changes in a runner's pace relative the average amount by which they deviated from the empirical 24-hour pace.

Chart 13, which combines both genders, shows a distinct pattern. Top 10 runners accelerate throughout the race by about 1.5 seconds per mile per mile (which accumulates to over 2.5 minutes per mile over 100 miles). In other words, the twenty Top 10 finishers are not only averaging well below silver buckle pace, but they are actually accelerating relative to this pace throughout the race by about 1.5 seconds per mile, every mile. The one clear exception to this rule is Andrea Huser. Unlike the other 19, she did not accelerate over the course of the race. She exceeded 24-hour pace by about the same amount throughout the race.

The Sub-24 Hour finishers also accelerate, though not by as much. In this case, the acceleration is about 0.7 seconds per mile per mile, which is about half the acceleration of the Top 10 finishers. Still, this accumulates to a speed-up of 1 minute and 11 seconds per mile over the course of 100 miles.
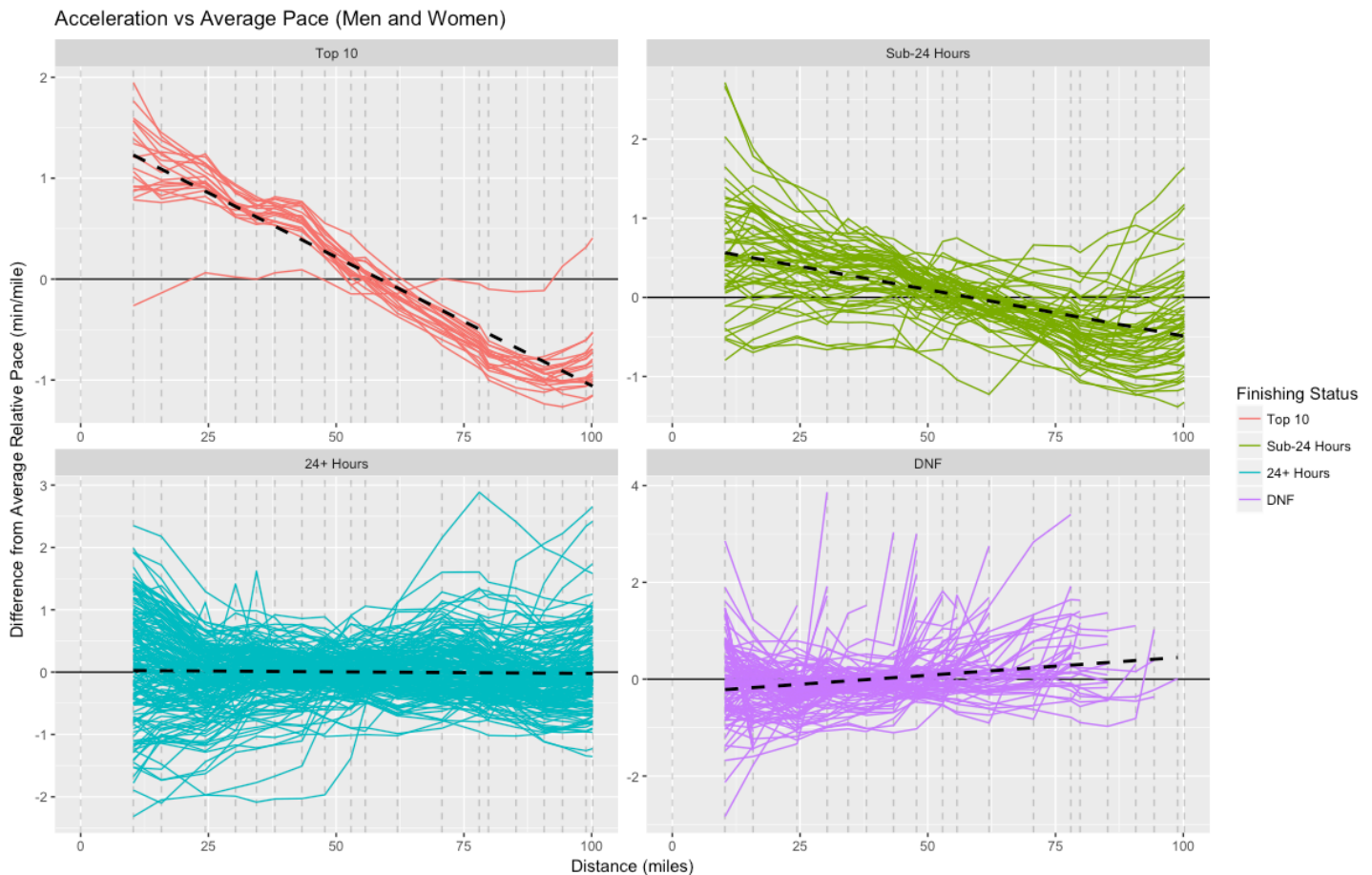
Chart 13: *Runner pace relative to average relative pace*

On the other hand, non-silver buckle winners do not accelerate at all. The 24+ Hour finishers were essentially flat throughout the race and the DNFs as a group actually slowed down by almost a half second per mile per mile. Table 6 summarizes these findings for the four finishing categories.
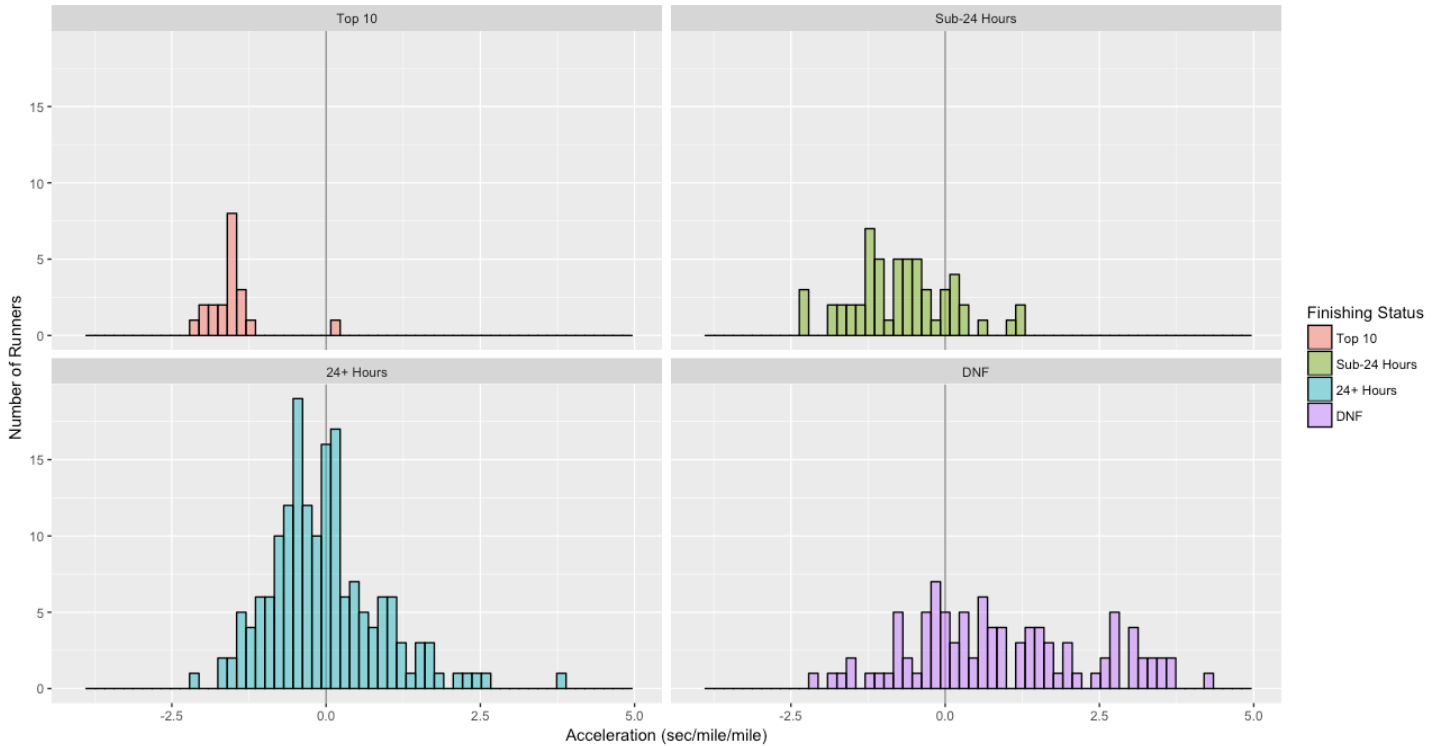
Status	Acceleration (sec/mile/mile)	Total Acceleration (min/mile)
Top 10	1.53	2:33
Sub-24 Hours	0.70	1:11
24+ Hours	0.03	0:03
DNF	-0.45	-1:15

Table 6: *Acceleration speed by finishing status*

From this data we can conclude that, on average, starting slowly and gradually accelerating is the smart way to earn a silver buckle. Those runners that simply maintain their pace or slow down do not typically fare well.

Chart 14 shows a histogram of all runners and their acceleration in seconds per mile per mile. You can again clearly see Andrea Huser as the exception among Top 10 finishers for not accelerating. In fact, 63 of the 76 silver buckle winners accelerated over the course of the race (83%). On the other hand, only 139 of the 292 non-silver buckle winners accelerated (48%).

Chart 14: *Runner acceleration histogram by finishing status*

Aid Station Spectator Guide

Until now, we've focused on pacing from a runner's perspective. In this section, we consider things from the perspective of a spectator at a given aid station. In particular, we ask the question: Given that I see a runner arrive under (or over) the empirical silver buckle pace, what are the chances they eventually earn (or fail to earn) a silver buckle?

To answer this question, we can utilize one of the core theorems of probability theory: Bayes' Theorem. Mathematically, Bayes' Theorem states:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In our case, we can let *A* represent "earns silver buckle" and *B* represent "arrived ahead of 24-hour pace". In that case, we have:

$$P(silver\ buckle|ahead\ of\ pace) = \frac{P(ahead\ of\ pace|silver\ buckle)\,P(silver\ buckle)}{P(ahead\ of\ pace)}$$

In words, the likelihood of a runner earning a silver buckle given that they are ahead of pace at a given aid station is equal to the likelihood that they are ahead of pace given that they earned a silver buckle, times the likelihood of earning a silver buckle, divided by the likelihood of being ahead of pace.

Let's look at the concrete example of a spectator at Foresthill (mile 62). Suppose we observe a runner arrive ahead of our model's silver buckle pace, 13:56. Then what is the likelihood that they eventually earn a silver buckle? For the Foresthill aid station, we have the following runner counts:

	Ahead of Pace	Behind Pace	Total
Silver Buckle	52	24	76
No Silver Buckle	10	215	225
Total	62	239	301

Therefore,

$$P(ahead\ of\ pace) = \frac{62}{301} \approx 20.6\%$$

$$P(silver\ buckle) = \frac{76}{301} \approx 25.2\%$$

$$P(ahead\ of\ pace|silver\ buckle) = \frac{52}{76} \approx 68.4\%$$

And finally,

$$P(silver\ buckle|ahead\ of\ pace) = \frac{{52}/{76} \cdot {76}/{301}}{{62}/{301}} = \frac{26}{31} \approx 83.9\%$$

Therefore, when we observe a runner pass through Foresthill under 13:56, we can be approximately 84% confident that they will go on to earn the coveted silver buckle. Conversely, there is a 16% likelihood they will not.

If we repeat this process for each of the aid stations (and expand from two categories of finishing status to four), we arrive at the data presented in Chart 15. The left chart shows the likelihood of each finishing status for those runners arriving over the empirical silver buckle pace. The right chart shows the same for those runners arriving under the empirical pace.
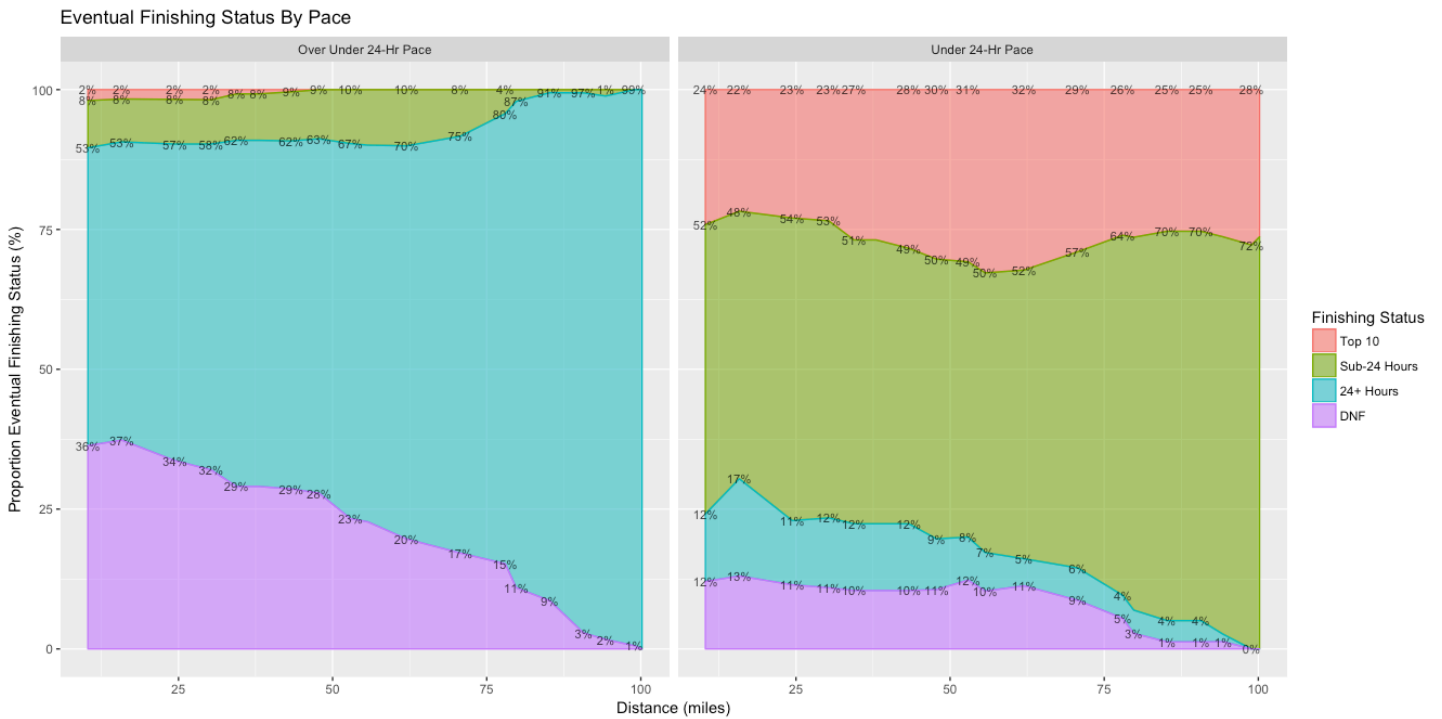
Chart 15: *Conditional likelihood of finishing status by aid station and pace*

A few things become apparent with these charts. First, for those runners arriving at the first aid station (Lyon Ridge, mile 10.3) behind pace, there is around a 36% likelihood of an eventual DNF (shown in purple). This likelihood steadily declines as those runners exit the race. We see a "survival of the fittest" pattern emerge whereby the field grows

stronger as the race progresses and the likelihood of a DNF declines with each mile. Interestingly, however, the likelihood of a DNF for those runners arriving ahead of silver buckle pace holds steady around 10% all the way to Peachstone (mile 70.7).

Second, for those runners arriving behind silver buckle pace, there is about a 10% chance of rallying to an eventual silver buckle. This 10% likelihood exists all the way to the Foresthill (mile 62) aid station. After that, however, the likelihood rapidly shrinks to near zero at Auburn Lake Trails (mile 85.2). In other words, for a small portion of the field, rallying from behind 24-hour pace to a silver buckle finish is possible but the rally needs to begin by Foresthill. Starting any later and the likelihood rapidly diminishes.

The Silver Buckle Frontier

In this section we investigate the question: What is the farthest behind 24-hour pace a runner can be and still earn a silver buckle?

Recall that our empirical silver buckle pace represented the pace such that the likelihood of an eventual silver buckle finish was exactly 50%. Thus, it is likely that there will be a subset of the field that runs behind model pace for a portion of the race but finishes strong and nonetheless earns a silver buckle. How far back were these runners?

First, let's examine the data visually. Chart 16 shows the elapsed time of each male runner relative to the empirical 24-hour pace. The most relevant chart is the one for Sub-24 Hour runners. Of the 51 runners in this category, 25 fell behind 24-hour pace at some point yet rallied to finish under 24 hours. (None of the top 10 men were ever behind silver buckle pace.) Conversely, for the 141 runners who finished in more than 24 hours, only 11 were ever ahead of silver buckle pace at any point.
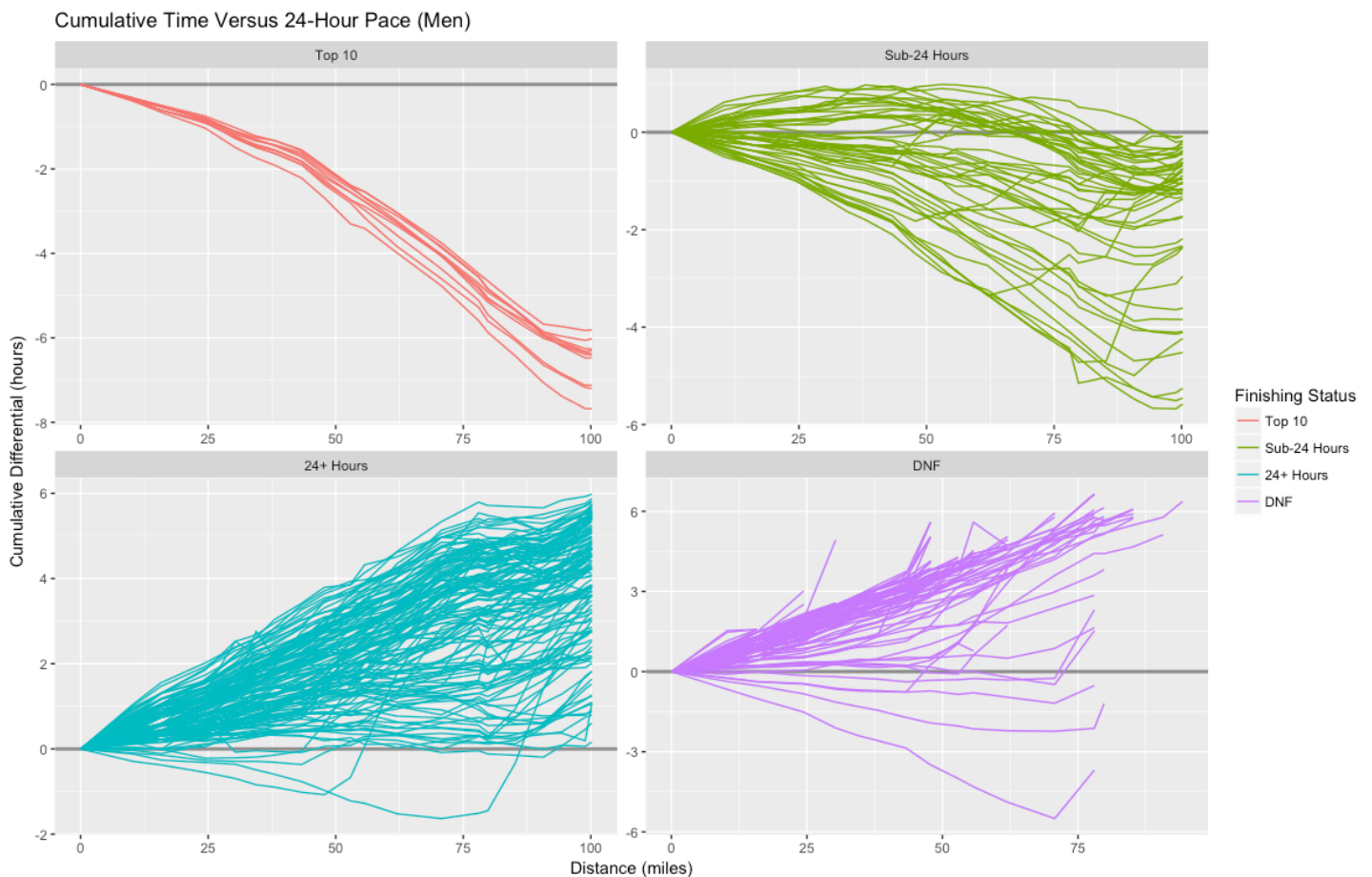
Chart 16: *Elapsed time relative to silver buckle pace (men)*

Chart 17 shows the same for the women. Unlike the men, six of the top ten women were behind pace at some point. Additionally, four of the five women who earned a silver buckle outside the top ten were behind pace by at least 30 minutes.
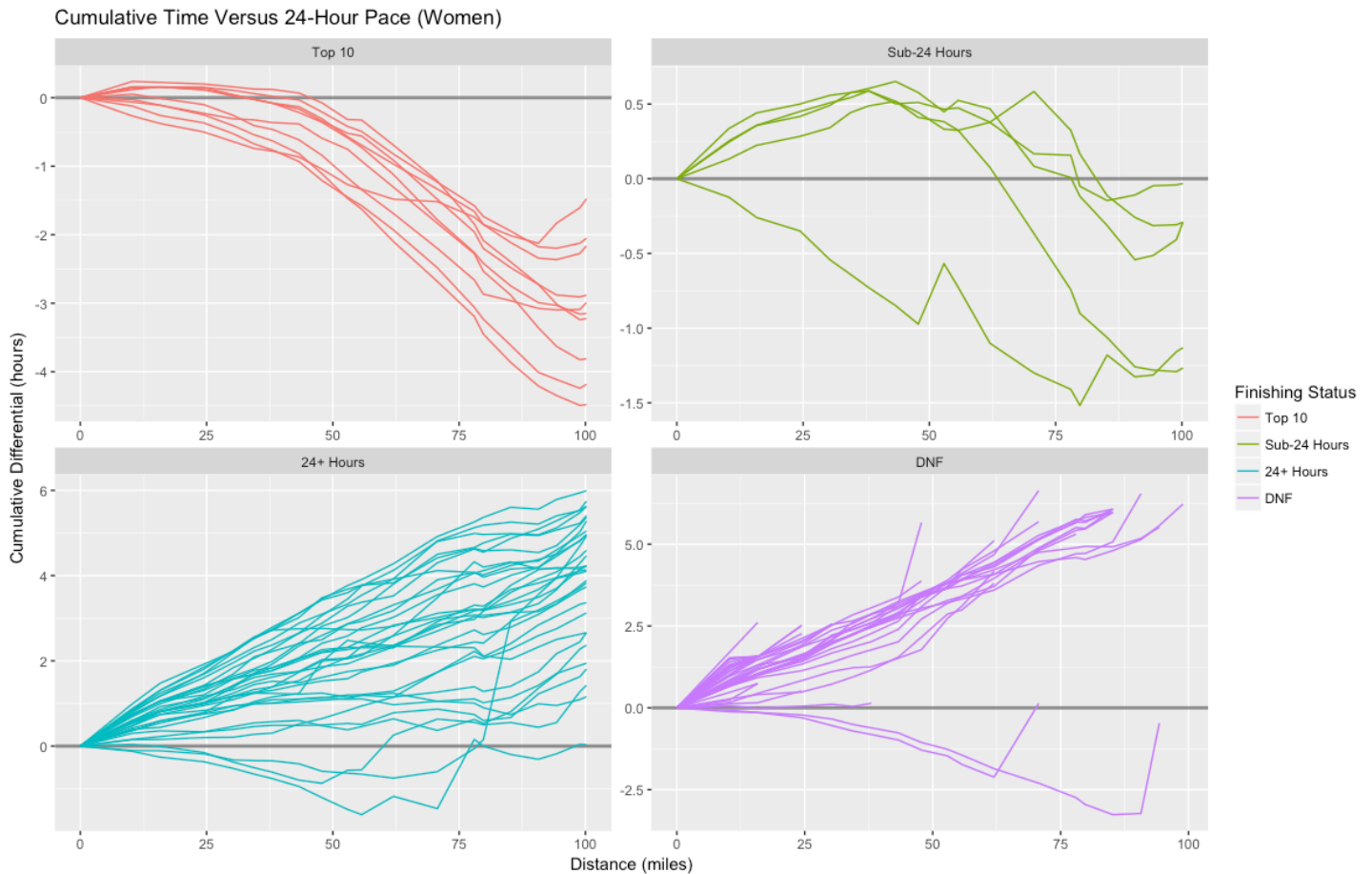
Chart 17: *Elapsed time relative to silver buckle pace (women)*

We will define the "silver buckle frontier" at a particular aid station as the elapsed time (to that aid station) of the final runner who reached that age station and still went on to earn a silver buckle. In other words, it's the size of the biggest comeback from that aid station. By definition, any runner arriving after the "silver buckle frontier" did not earn a silver buckle.

Table 7 shows the silver buckle frontier for each aid station. You can see that some runners were behind pace by as much as 58 minutes between Dusty Corners (mile 38) and Michigan Bluff (mile 55.7).

Aid Name	Aid Distance	Frontier
Start	0.0	+0:00
Lyon Ridge	10.3	+0:37
Red Star Ridge	15.8	+0:44
Duncan Canyon	24.4	+0:50
Robinson Flat	30.3	+0:56
Miller's Defeat	34.4	+0:53
Dusty Corners	38.0	+0:58
Last Chance	43.3	+0:57
Devil's Thumb	47.8	+0:56
El Dorado Creek	52.9	+0:59
Michigan Bluff	55.7	+0:58
Foresthill	62.0	+0:54
Peachstone	70.7	+0:43
Rucky Chucky	78.0	+0:38
Green Gate	79.8	+0:31
Auburn Lake Trails	85.2	+0:26
Quarry Road	90.7	+0:15
Pointed Rocks	94.3	+0:02
Robie Point	98.9	-0:02
Finish Line	100.2	-0:02

Table 7: *Largest comeback by aid station*

To make the frontier more usable, we will add 5 minutes to each point (other than the start and finish lines, which we set to zero) and "smooth it out" by fitting a polynomial through the points. In this case, we'll use a degree 5 polynomial. Chart 18 shows the elapsed time (relative to 24-hour pace) for all men who were ahead of pace at any point, with the silver buckle frontier overlaid. You can see that the frontier does indeed contain beneath it all the points.

By definition, all of the silver buckle winners stayed under this frontier. Conversely, all of the non-silver buckle winners either crossed the frontier (and therefore failed to finish under 24 hours) or dropped out without ever crossing it. As it happens, there were three men who never crossed the frontier yet failed to earn a silver buckle: Jim Walmsley, David Byrne and Nate Jaqua. (These are the bottom three purple lines in the chart.)
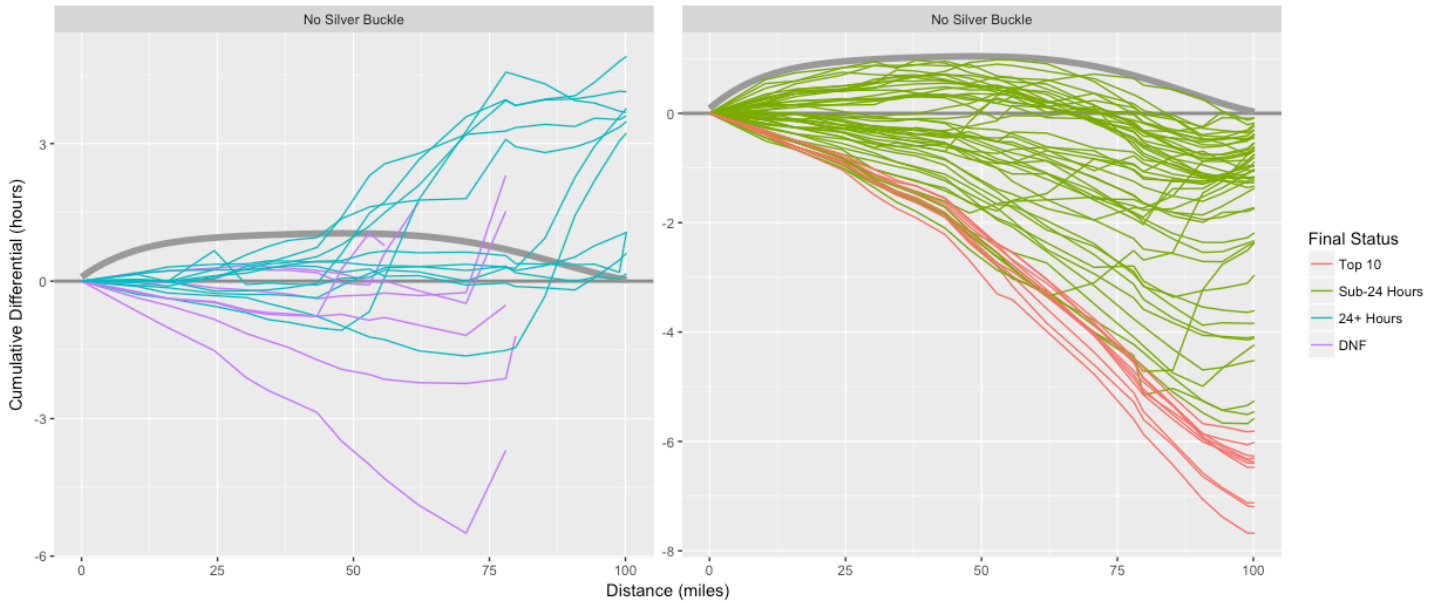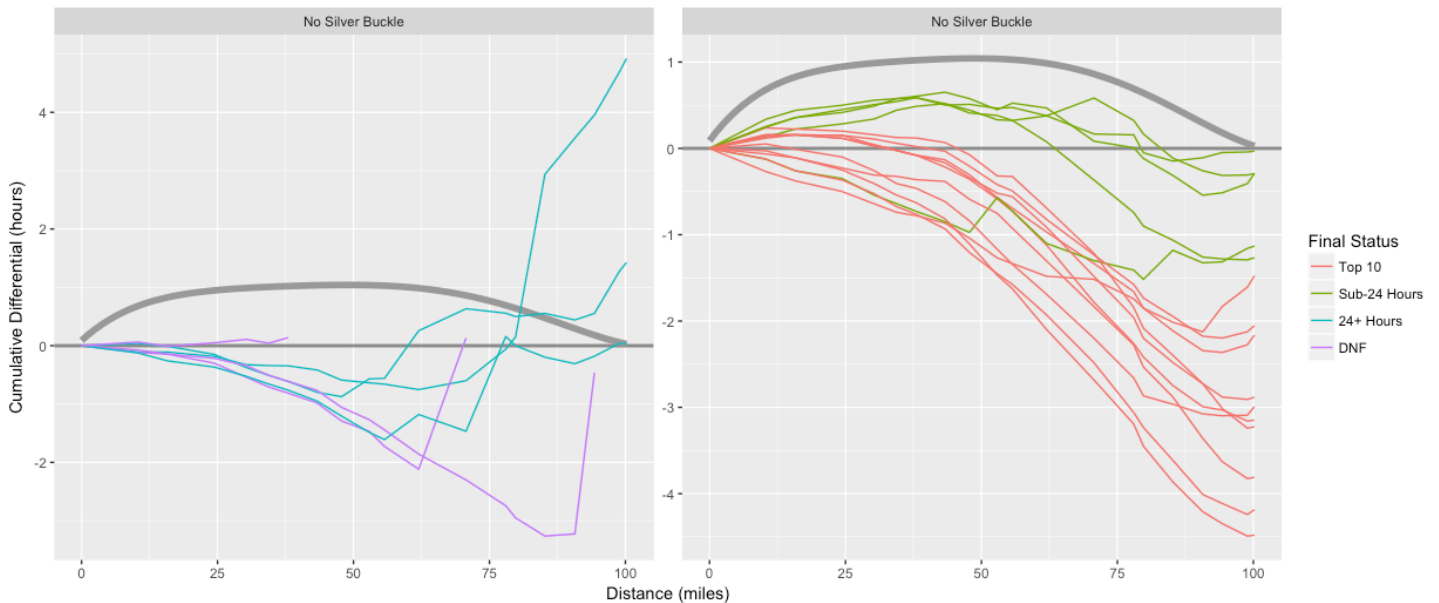
Chart 18: *Elapsed time relative to silver buckle pace with frontier (men)*

Chart 19 shows the same thing for women. Unlike the men, the female silver buckle winners all stayed well under the frontier throughout the race. However, like on the men's side, there were three women who never crossed the frontier yet failed to earn a silver buckle: Clare Gallagher, Yiou Wang and Amy Sproston.

Chart 19: *Elapsed time relative to silver buckle pace with frontier (men)*

Table 8 shows the complete pace chart with the official pace, the empirical pace built by our model, the buffer between the empirical pace and the silver buckle frontier and finally the frontier pace. As an example of how to read this table, consider a runner arriving at the Peachstone (mile 70.7) aid station. The official 24-hour pace to this point is 15:45. However, based on actual experience, our model estimates that a runner arriving 22 minutes later still has a 50% likelihood of earning a silver buckle. Furthermore, a runner arriving a full 71 minutes later can still rally to a silver buckle!

Aid Name	Aid Distance	Official Pace	Empirical Pace	Buffer	Frontier Pace
Start	0.0	0:00	0:00	+0:00	0:00
Lyon Ridge	10.3	2:10	2:16	+0:37	2:52
Red Star Ridge	15.8	3:20	3:26	+0:47	4:12
Duncan Canyon	24.4	5:00	4:59	+0:57	5:56
Robinson Flat	30.3	6:30	6:27	+1:01	7:28
Miller's Defeat	34.4	7:15	7:12	+1:03	8:15
Dusty Corners	38.0	7:55	7:50	+1:04	8:53
Last Chance	43.3	8:55	8:45	+1:04	9:49
Devil's Thumb	47.8	10:15	10:09	+1:03	11:12
El Dorado Creek	52.9	11:20	11:20	+1:01	12:21
Michigan Bluff	55.7	12:20	12:19	+1:00	13:18
Foresthill	62.0	13:45	13:56	+0:56	14:52
Peachstone	70.7	15:45	16:07	+0:49	16:56
Rucky Chucky	78.0	17:40	18:02	+0:41	18:42
Green Gate	79.8	18:20	18:45	+0:39	19:24
Auburn Lake Trails	85.2	19:50	20:14	+0:31	20:44
Quarry Road	90.7	21:10	21:43	+0:21	22:03
Pointed Rocks	94.3	22:20	22:43	+0:13	22:56
Robie Point	98.9	23:40	23:46	+0:02	23:49
Finish Line	100.2	24:00	24:00	+0:00	24:00

Table 8: Pace chart with silver buckle frontier

Biggest Comebacks

In this section, we identify those silver buckle winners who made the largest comeback (relative to empirical 24-hour pace). First, Table 9 shows the top 5 biggest male comebacks. Richard Snipes (age 42 from San Anselmo, CA) was 59 minutes behind at El Dorado Creek (mile 52.9) yet rallied for a silver buckle in 23:49. Even more improbable, our model gave Ian Seabury (age 27 from Los Angeles, CA) just a 13% chance of earning a silver buckle at Dusty Corners (mile 38) after being 58 minutes behind pace, yet he finished well under 24 hours with more than 40 minutes to spare.

Name	Max Differential	Silver Buckle Probability	Aid Station	Aid Distance	Finish
Richard Snipes	+0:59	18%	El Dorado Creek	52.9	23:49
Ian Seabury	+0:58	13%	Dusty Corners	38.0	23:19
Jonathan Faryadi	+0:57	15%	Last Chance	43.3	23:20
Karl Hoagland	+0:54	18%	Devil's Thumb	47.8	23:28
Kim Espat	+0:48	15%	Miller's Defeat	34.4	22:58

Table 9: Biggest comebacks (by maximum differential) for men

Table 10 shows the same thing for women. The largest comeback was Kristy McBride (age 31 from Charlevoix, MI) who was 39 minutes behind at Last Chance (mile 43.3). Of particular note is Jacqueline Merritt (age 29 from Atlanta, GA) who started very conservatively and was 14 minutes behind silver buckle pace at the first aid station, Lyon Ridge (mile 10.3). From there, however, she ran to ran to a strong 7th place finish and a well-earned silver buckle in 21:07.

Name	Max Differential	Silver Buckle Probability	Aid Station	Aid Distance	Finish
Kristy McBride	+0:39	24%	Last Chance	43.3	23:42
Mallory Richard	+0:35	24%	Dusty Corners	38.0	22:44
Mandie Holmes	+0:35	24%	Dusty Corners	38.0	23:42
Stephanie Case	+0:31	28%	Last Chance	43.3	23:58
Jacqueline Merritt	+0:14	19%	Lyon Ridge	10.3	21:07

Table 10: Biggest comebacks (by maximum differential) for women

Biggest Blowups

In this section, we identify those runners who were ahead of the empirical 24-hour pace by the most yet failed to earn a silver buckle. Table 11 shows the top 5 male blow ups. Not surprisingly, Jim Walmsley tops the list with a buffer of nearly five and a half hours under silver buckle pace at Peachstone (mile 70.7). Our model gave David Byrne a 95% chance of a silver buckle after coming into Peachstone with a buffer of over two hours. Like Walmsley, however, David later dropped. Jason Mintz (age 37 from Syracuse, NY) and Joel Frost-Tift (age 29 from Huntington Park, CA) gave up buffers of more than an hour but stuck it out and finished the race.

Name	Max Differential	Silver Buckle Probability	Aid Station	Aid Distance	Finish
Jim Walmsley	-5:29	100%	Peachstone	70.7	DNF
David Byrne	-2:13	95%	Peachstone	70.7	DNF
Jason Mintz	-1:37	90%	Peachstone	70.7	27:13
Nate Jaqua	-1:10	83%	Peachstone	70.7	DNF
Joel Frost-Tift	-1:04	86%	Devil's Thumb	47.8	27:41

Table 11: Biggest blowups (by maximum differential) for men

Table 12 shows the same for women. Clare Gallagher was more than three hours ahead of silver-buckle pace and in third place at Auburn Lake Trails (mile 85.2) but later dropped. Another notable blowup was Yiou Wang who arrived at Foresthill (mile 62) in the lead and more than two hours ahead of 24-hour pace but also later dropped.

Name	Max Differential	Silver Buckle Probability	Aid Station	Aid Distance	Finish
Clare Gallagher	-3:15	100%	Auburn Lake Trails	85.2	DNF
Yiou Wang	-2:07	95%	Foresthill	62.0	DNF
Kaci Lickteig	-1:36	92%	Michigan Bluff	55.7	24:02
Emily Harrison	-0:52	81%	Devil's Thumb	47.8	25:25
Sarah Keyes	-0:45	75%	Foresthill	62.0	28:55

Table 12: Biggest blowups (by maximum differential) for women

Steadiest Performance

In this section, we identify those runners who earned a silver buckle with the smallest deviation from the empirical 24-hour pace. We will measure this by the range between each runner's largest deviation above and below pace. These runners stayed on pace and steady all day, arriving in Auburn just under the cutoff. Table 13 shows the top five steadiest performances for men. Angel Ochoa (age 29 from Tucson, AZ) was 21 minutes behind pace at Dusty Corners (mile 38) and 14 minutes ahead of pace at Quarry Road (mile 90.7), ultimately finishing with 11 minutes to spare in 23.49.

Name	Max	Min	Range	Finish
Angel Ochoa	+0:21	-0:14	0:35	23:49
Matt Zuchetto	+0:08	-0:31	0:39	23:34
Paulo Medina	+0:18	-0:23	0:41	23:46
Andrew Stevens	+0:43	-0:07	0:50	23:55
Philip Sanderson	+0:30	-0:19	0:50	23:55

Table 13: *Steadiest silver-buckle performance for men*

Table 14 shows the same for women. Stephanie Case (age 35 from Geneva, Switzerland) was 31 minutes behind pace at Last Chance (mile 43.3) and 8 minutes ahead of pace at Auburn Lake Trails (mile 85.2). She become the final silver buckle winner in 23:58. Also noteworthy are the performances of Mandie Holmes (age 31 from Mountain View, CA) and Mallory Richard (age 32 from Winnipeg, Canada) who both finished hard and reached their peak differential at the finish line.

| Name	Max	Min	Range	Finish
Stephanie Case	+0:31	-0:08	0:39	23:58
Mandie Holmes	+0:35	-0:18	0:54	23:42
Kristy McBride	+0:39	-0:32	1:11	23:42
Stephanie Howe Violett	+0:00	-1:31	1:31	22:52
Mallory Richard	+0:35	-1:17	1:52	22:44

Table 14: *Steadiest silver-buckle performance for women*

Most Unconventional Pacing

In this section, we examine the very unconventional pacing strategy of one particular runner: Andrew Stevens (age 40 from Stourport, England). Andrew got off to a strong start and arrived at Lyon Ridge (mile 10.3) seven minutes ahead of 24-hour pace. Our model gave him a 67% chance of a silver-buckle at that early point. From there, however, things slowly deteriorated. He gave back time at each of the next nine aid stations, eventually arriving at Peachstone (mile 70.7) a full 43 minutes behind pace. From here he began to rally, but not fast enough for our model to improve his likelihood of a silver buckle. In fact, as late as Auburn Lake Trails (mile 85.2), the model gave him just a 22% likelihood of a sub-24-hour finish. However, he made up a deficit of 26 minutes in only 15 miles and finished in 23:55.

Chart 20 shows Andrew's likelihood of finishing under 24 hours at each aid station (in red) compared with the other 75 silver buckle winners (in gray). You can see that he is a clear outlier. In fact, he was the last of the eventual silver buckle winners to arrive at Auburn Lake Trails (mile 85.2) and did so a full 28 minutes the other 75 runners!
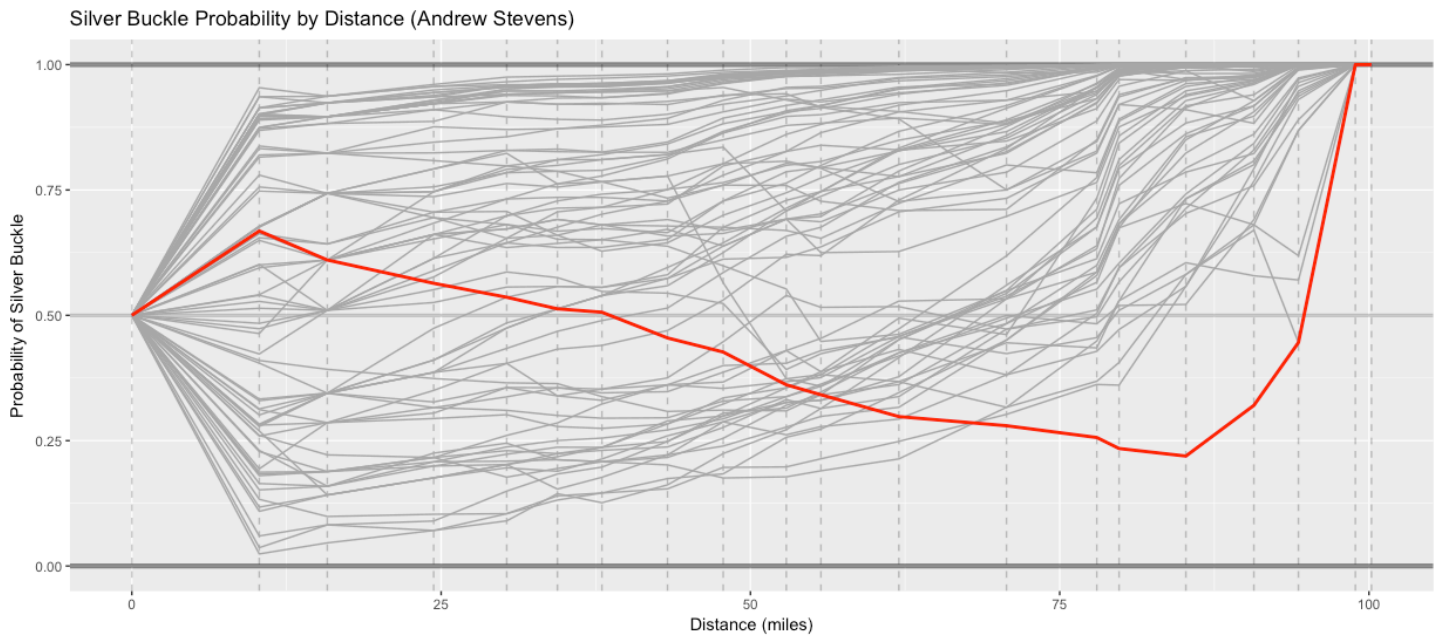
Chart 20: *Silver buckle probability for Andrew Stevens*

Conclusion

In this paper, we have done several things. First, we used the actual results of the 2017 race to build a model of 24-hour pace which best separated silver-buckle winners from non-winners at each of the 18 aid stations. This revealed that the official 24-hour pace was generally reasonable, but was too fast by as many as 32 minutes late in the race. We also calculated a "frontier" pace which estimated the limit from which a silver-buckle comeback was still possible. We demonstrated that runners could be up to 71 minutes behind the official pace and still rally to a sub-24-hour finish.

Using our model, we examined the pacing strategies of various groups of runners. We discovered that silver buckle winners consistently accelerate their pace throughout the race while non-winners do not. Lastly, we looked at many examples of runners that had noteworthy pacing strategies, many of whom rallied from significant deficits to achieve their goal of a Western States silver buckle.

The bottom line: start slow, don't worry about the official 24-hour cutoffs and gradually accelerate all the way to the finish. See you on the track in Auburn!